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SCALING STUDIES OF QCD ON ASYMMETRIC LATTICES †

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ABSTRACT

Using the deconfinement temperature as a probe, the scaling properties of QCD are studied on asymmetric lattices. We find that the measured asymmetry dependence of the lattice Λ parameter at $\beta \approx 5.7$ agrees precisely with that predicted by one loop perturbation theory. This result holds on lattices with asymmetry in one spatial direction and on lattices with asymmetry in all three spatial directions and suggests that the perturbative scaling violations observed on symmetric lattices at these values of β are independent of asymmetry and therefore unlikely to be lattice artifacts.

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Using the deconfinement temperature as a probe, the scaling properties of QCD are studied on asymmetric lattices. We find that the measured asymmetry dependence of the lattice Λ parameter at $\beta \approx 5.7$ agrees precisely with that predicted by one loop perturbation theory. This result holds on lattices with asymmetry in one spatial direction and on lattices with asymmetry in all three spatial directions and suggests that the perturbative scaling violations observed on symmetric lattices at these values of β are independent of asymmetry and therefore unlikely to be lattice artifacts.

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An understanding of the scaling properties of lattice QCD is of prime importance in relating the results of numerical calculations to the the continuum physics those calculations hope to investigate. Studies of scaling have therefore received significant attention in the recent literature and some considerable progress has been achieved for the pure (*i.e.* without fermions) QCD model. However, there are still a number of issues pertaining to scaling and universality in pure gauge QCD which have not been clarified.

The scaling region in pure lattice QCD is, by definition, that region of coupling constant space in which all dimensionless ratios of dimensionful physical observables become independent of the coupling constant. More explicitly, consider how a dimensionful physical observable, for example a mass M, is measured on the lattice. We find that

$$M = F_M(g)/a \tag{1}$$

where a is the lattice spacing and $F_M(g)$ is a measurable function of the coupling g. Requiring that M be independent of the lattice spacing now forces us to choose the coupling to be a function of the lattice spacing (*i.e.* $g \equiv g(a)$). For general couplings each different physical observable will require a different choice for this function. In the scaling region, however, there exists a single unique choice (up to an integration constant) for $g(a)$ which leaves all possible low momentum physical observables independent of lattice spacing.

In pure QCD the scaling region includes the point $g = 0$ and close to this point the function $g(a)$ satisfies the perturbative renormalization group equation [1]

$$a\frac{dg}{da} = -\beta(g) = b_0 g^3 + b_1 g^5 + \ldots \tag{2}$$

which has the solution

$$\Lambda a = \exp\left(-\frac{1}{2b_0 g^2} - \frac{b_1}{2b_0^2}\ln\left(b_0 g^2\right)\right) \tag{3}$$

Λ is the constant of integration which defines the physical scale for the theory. Recently Gottlieb *et al.* and Christ and Terrano [2] have shown that the deconfinement temperature scales according to this perturbative equation in the region $6.1 \lesssim 6/g^2 \lesssim 6.5$. This is encouraging and suggests that perturbative or asymptotic scaling sets in at $6/g^2 \gtrsim 6.1$.

For $6/g^2 \lesssim 6.1$ the picture is not so clear. There is evidence from measurements of the deconfinement temperature and the string tension, and from renormalization group calculations [3] that nonperturbative scaling holds in the region $5.6 \lesssim 6/g^2 \lesssim 6.1$. Glueball mass calculations in this region are also consistent with non perturbative scaling [4] but the error bars on the glueball results are large enough that a definitive statement is not possible. If scaling does indeed hold in this region then continuum lattice calculations become possible at $6/g^2 \approx 5.7$ resulting in large savings in the computer resources required to do any given calculation. More importantly even if scaling doesn't hold in this region it would be comforting to have some understanding of exactly what is causing the breakdown in perturbative scaling observed here. Thus lattice studies of pure QCD in this region are of great interest.

There is a another aspect of scaling which also needs further investigation. This arises from the fact that there are many different possible lattice actions which give rise to continuum QCD in the limit $a \to 0$. These actions include the Wilson action, the fundamental-adjoint action [5], the Manton action [6] and the Villain action [7]. Also the lattice on which each of these actions is embedded need not always be the standard hyper-cubic lattice. Hyper-rectangular (asymmetric) [8,9] or indeed random lattices [10] can all be used. Each of these various possible lattice actions will have their own scaling regions which will include the point $g = 0$ together with a region of perturbative scaling about this point. It is important to check that these various choices give rise to the same continuum physics.

Some work has been already be done on this question. Patel *et al.* have made measurements of the ratio of the glueball mass to the string tension for a lattice action which includes 1×1 plaquettes in the fundamental, 6 and 8 representations of $SU(3)$ and also 1×2 plaquettes [11]. The relevant coupling for these calculations was $6/g^2 \approx 5.9$. The results obtained are significantly different from similar results obtained for the Wilson action. This suggests that universality, in the sense that different lattice actions are giving the same physics, has not yet set in at these values of $6/g^2$.

More recently Toussaint and Buendia [12] have looked at the universality of the deconfinement temperature for various choices of fundamental-adjoint lattice actions for values of $6/g^2 \approx 6$. The results of this work suggest that universality may be present at least for a subset of actions close to the Wilson action. However as the action studied varies further from the pure Wilson action universality appears to break down.

Concurrently with the work of Toussaint we have been studying universality on asymmetric lattices [9]. Specifically we have chosen to examine the deconfinement temperature on lattices with spacing $a_\mu = \xi_\mu a$ along the μ^{th} axis. The variables ξ_μ introduced here are dimensionless asymmetry parameters. In the perturbative scaling region different choices for the asymmetry parameters represent different lattice regularizations and all asymmetry dependences in the theory should therefore be removable by appropriate redefinitions of the scale parameter Λ.

The pure gauge $SU(N)$ asymmetric action which we have chosen to use is given by [9,10]

$$S(U) = \sum_{x,\mu<\nu} \frac{6}{g^2} \frac{\xi_x \xi_y \xi_z \xi_t}{\xi_\mu^2 \xi_\nu^2} \left(1 - \frac{1}{3}\mathrm{ReTr}\left(U_{\mu\nu}(x)\right)\right) \tag{4}$$

where $U_{\mu\nu}(x)$ is as usual the path ordered product of 3×3 unitary link matrices about the fundamental plaquette in the μ-ν plane at the point x.

For general choices of the asymmetries ξ_μ there is insufficient symmetry in the lattice action of Eqn.(4) to guarantee that a Lorentz invariant continuum theory is recovered as $a \to 0$. However, when three of the four asymmetry parameters are equal then the resulting three dimensional cubical symmetry on the lattice is sufficient to enforce Lorentz symmetry in the continuum. In our case, since we wish to study the deconfinement temperature, the time direction of the lattice has special significance. There are therefore two distinct classes of asymmetric lattices which we can study. The first of these has asymmetry along the x-axis only. The second has equal asymmetry along each of the three spatial axes. Specifically the two classes are

$$\begin{aligned} &\text{Class 1}: \quad \xi_x = \xi_{\text{space}}, \quad \xi_y = \xi_z = \xi_t = 1 \\ &\text{Class 2}: \quad \xi_x = \xi_y = \xi_z = \xi_{\text{time}}, \quad \xi_t = 1 \end{aligned} \tag{5}$$

For various choices of ξ_{space} and ξ_{time} we have measured the critical coupling $((6/g^2)_c)$ at which deconfinement occurs on lattices with two and four sites in the time direction.

Before discussing the Monte Carlo results let us briefly mention the behavior that perturbative scaling implies. In one loop lattice perturbation theory it is a relatively simple matter to calculate how $(6/g^2)_c$ should change with asymmetry. One uses standard background field methods to relate the relevant Λ parameters. The result of these calculations is an expression for the difference, $\Delta(6/g^2)$, of the coupling constants at different asymmetries and lattice spacings which takes the form

$$\Delta\left(\frac{6}{g^2}(\xi,\xi_0;a,a_0)\right) = \frac{6}{g^2}(\xi,a) - \frac{6}{g^2}(\xi_0,a_0) = B(a/a_0) + C(\xi,\xi_0) \tag{6}$$

$B(a/a_0)$ depends only on the relevant lattice spacings and represents the finite difference of infrared divergent Feynmann integrals. $C(\xi,\xi_0)$, on the other hand, is independent of the lattice spacing. For further details we refer the reader to references [9] and [13]. The aim of our Monte Carlo calculations will be to test how well these one loop scaling results hold in the region $6/g^2 \lesssim 6.0$.

In order to determine the critical coupling $(6/g^2)_c$ at deconfinement we used the Polyakov line operator $P(\vec{x})$ which is defined to be

$$P(\vec{x}) = \mathrm{Tr}\left(U_t(\vec{x})U_t(\vec{x}+\hat{t})\ldots U_t(\vec{x}+N_t\hat{t})\right) \tag{7}$$

where the matrices U_t are the time like link matrices and N_t is the number of sites in the time direction. The expectation value of $P(\vec{x})$ is an order parameter for the transition from confinement to deconfinement in pure QCD. In the confined phase $P(\vec{x})$ has expectation value zero. In the deconfined phase $P(\vec{x})$ has finite non zero expectation value. The procedure we have adopted to measure the deconfinement temperature therefore was to measure the expectation value of $|P(\vec{x})|$ at each Monte Carlo sweep. If the parameters of a given Monte Carlo run were close to the transition values then the sweep to sweep values obtained for $|P(\vec{x})|$ clustered in two peaks, one corresponding to confinement and centered at a small value of $|P(\vec{x})|$, and one corresponding to deconfinement and centered at a larger value of $|P(\vec{x})|$. By counting the number of sweeps under each peak we can catalogue the percentage of sweeps of any given run which are confined. Then to determine the critical coupling for any given value of asymmetry and lattice size we generate from 20,000 to 50,000 sweeps at couplings close to the transition and interpolate to that value of $6/g^2$ for which 50% of sweeps are confined.

The results of our Monte Carlo studies are summarized in the three figures included here. In each case the figures show how the critical coupling at deconfinement varies as the asymmetry varies. For comparison each figure also contains the one loop perturbative predictions for the asymmetry dependence of $(6/g^2)_c$. These perturbative results were obtained from the analytic forms given in reference [9]. Let us now consider the three figures in turn.

In Figure 1 we have plotted the results obtained for lattices with two sites in the time direction as we vary the x-axis asymmetry ξ_{space}. Note first that the critical coupling for the symmetric lattice ($\xi_{space} = 1$) is $(6/g^2)_c = 5.069$. This is a value of coupling in the strong coupling sector of the theory far removed from the perturbative scaling region $6/g^2 \gtrsim 6.1$ and also far removed from the suggested nonperturbative scaling region $5.6 \lesssim 6/g^2 \lesssim 6.1$. Thus it is altogether understandable that the Monte Carlo data and the perturbative scaling predictions are in complete disagreement for this case.

In Figure 2 we show the results obtained for lattices with four sites in the time direction as the x-axis asymmetry changes. The symmetric lattice critical coupling for this case is $(6/g^2)_c = 5.676$ which is in the suggested nonperturbative scaling region but still far from the perturbative scaling sector. However, as the figure shows, we find that for $0.66 < \xi_{space} < 1.1$ there is surprisingly good agreement between the Monte Carlo data and the one loop perturbative predictions. Also note that the transitions from one loop behavior which occur at $\xi_{space} = .667$ and $\xi_{space} = 1.10$ appear to be very abrupt in nature.

Finally Figure 3 shows the results obtained for lattices with four sites in the time direction as ξ_{time} changes. As in Figure 2 we find a region of surprisingly good agreement between the Monte Carlo data and the one loop perturbative predictions. This region of agreement extends from $\xi_{time} = 0.85$ to $\xi_{time} = 1.03$. Note also that the transition from one loop behavior which occurs at $\xi_{time} = 1.03$ appears to be quite abrupt.

The results shown in Figures 2 and 3 pose two interesting questions. First, how can the asymmetry dependent perturbative agreement shown in these figures be consistent with the known nonperturbative behavior of the critical coupling on symmetric lattices when measured as a function of the lattice spacing a? Secondly, what is the origin of the apparently abrupt transitions from scaling which occur in both these figures?

Let us first address the question of the nature of the transition from scaling as the asymmetry changes. Note that it is not at all surprising, as the asymmetry increases, that lattice effects might set in and cause deviations from continuum scaling. Indeed such behavior is expected since increasing the asymmetry in a given direction increases the coarseness of the lattice in that direction. Thus the transitions occurring at $\xi_{\mathrm{space}} = 1.1$ in Figure 2 and at $\xi_{\mathrm{time}} = 1.03$ in Figure 3 are qualitatively reasonable. On the other hand it is initially surprising that similar transitions occur as the asymmetry decreases. One might expect that decreasing the asymmetry could only improve the continuum behavior of the theory since it makes the lattice grid finer in either one or all three spatial directions. (At the critical coupling, the lattice spacing in the temporal direction is by definition a fixed rational fraction of the inverse deconfinement temperature.)

A possible counter-argument arises, however, when one considers that the basic unit of lattice action is the trace of the product of link matrices about fundamental plaquettes. When the asymmetry gets large we find that in certain plaquettes one side gets large relative to the other. When the asymmetry gets small one side gets small relative to the other. Our results then suggest that the important considerations in determining whether a given asymmetry gives rise to continuum behavior are not only the overall size of the elementary lattice cell, but also the relative sizes of the sides in the fundamental plaquettes.

The more fundamental question which we need to address concerns the consistency between our result that the asymmetry dependence of the critical coupling is perturbative and the results of references [2] that the lattice spacing dependence of this same critical coupling is non perturbative at $6/g^2 \approx 5.7$. Our explanation for this phenomenon is based on the result summarized in Eqn.(6). This equation describes how the critical coupling changes both as the lattice spacing changes and as the asymmetry changes. Thus this equation is applicable both to the lattice spacing dependent scaling studies of Gottlieb et al. and of Christ and Terrano [2] and to the asymmetry dependent scaling studies described here. The important point to note, however, is that the two cases decouple. Eqn.(6) tells us that the lattice spacing dependence and the asymmetry dependence are completely independent of each other.

There is a further important point to be made. The term in Eqn.(6) which gives rise to the lattice spacing dependence of the theory $(B(a/a_0))$ receives contributions only from the infrared sector (*i.e.* only from momenta p which satisfy $p << \pi/a$). This explains how this term can be insensitive to the details of the lattice structure since these details are probed only by momenta on the order $p \approx \pi/a$. However this implies that the lattice spacing dependent term in Eqn.(6) is common to all lattice actions within the range for which perturbative asymmetry dependence is observed.

The scaling behavior exhibited in Figures 2 and 3 strongly suggests that the asymmetry dependence of the critical coupling at $6/g^2 = 5.7$ is given exactly by the term $C(\xi, \xi_0)$ in Eqn.(6). Thus our result suggests that the nonperturbative behavior observed at this value of coupling in reference [2] is infrared in nature and is confined to the term $B(a/a_0)$ only.

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FIGURE CAPTIONS

[1] Plot of the critical coupling $(6/g^2)_c$ as a function of asymmetry along the x-axis (ξ_{space}) for a lattice with two sites in the time direction. The solid line shown represents the prediction of one loop perturbation theory. The data points represent the Monte Carlo Results.

[2] Plot of the critical coupling $(6/g^2)_c$ as a function of asymmetry along the x-axis (ξ_{space}) for a lattice with four sites in the time direction.

[3] Plot of the critical coupling $(6/g^2)_c$ as a function of asymmetry along the three spatial directions (ξ_{time}) for a lattice with four sites in the time direction.

Figure 1

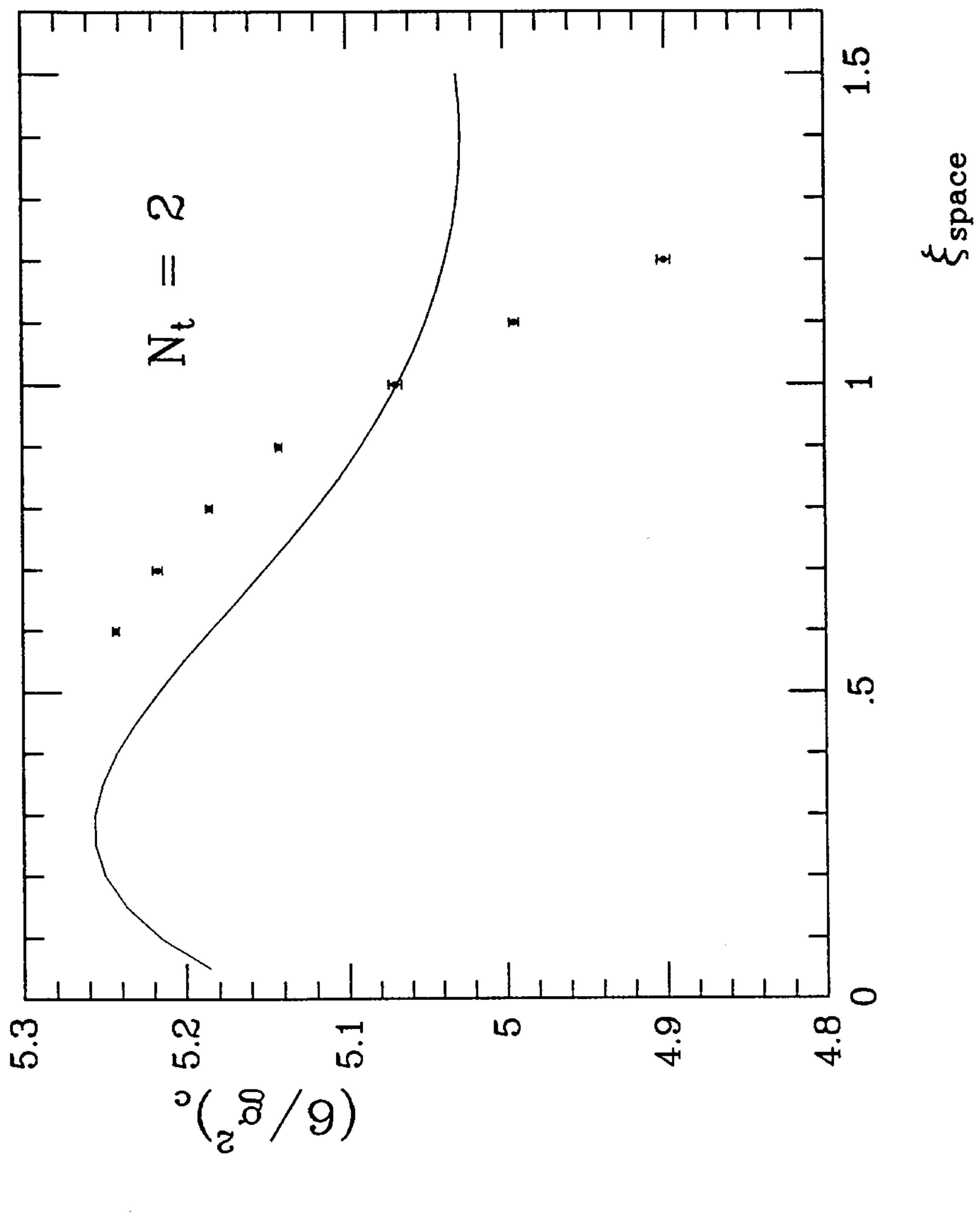

Figure 2

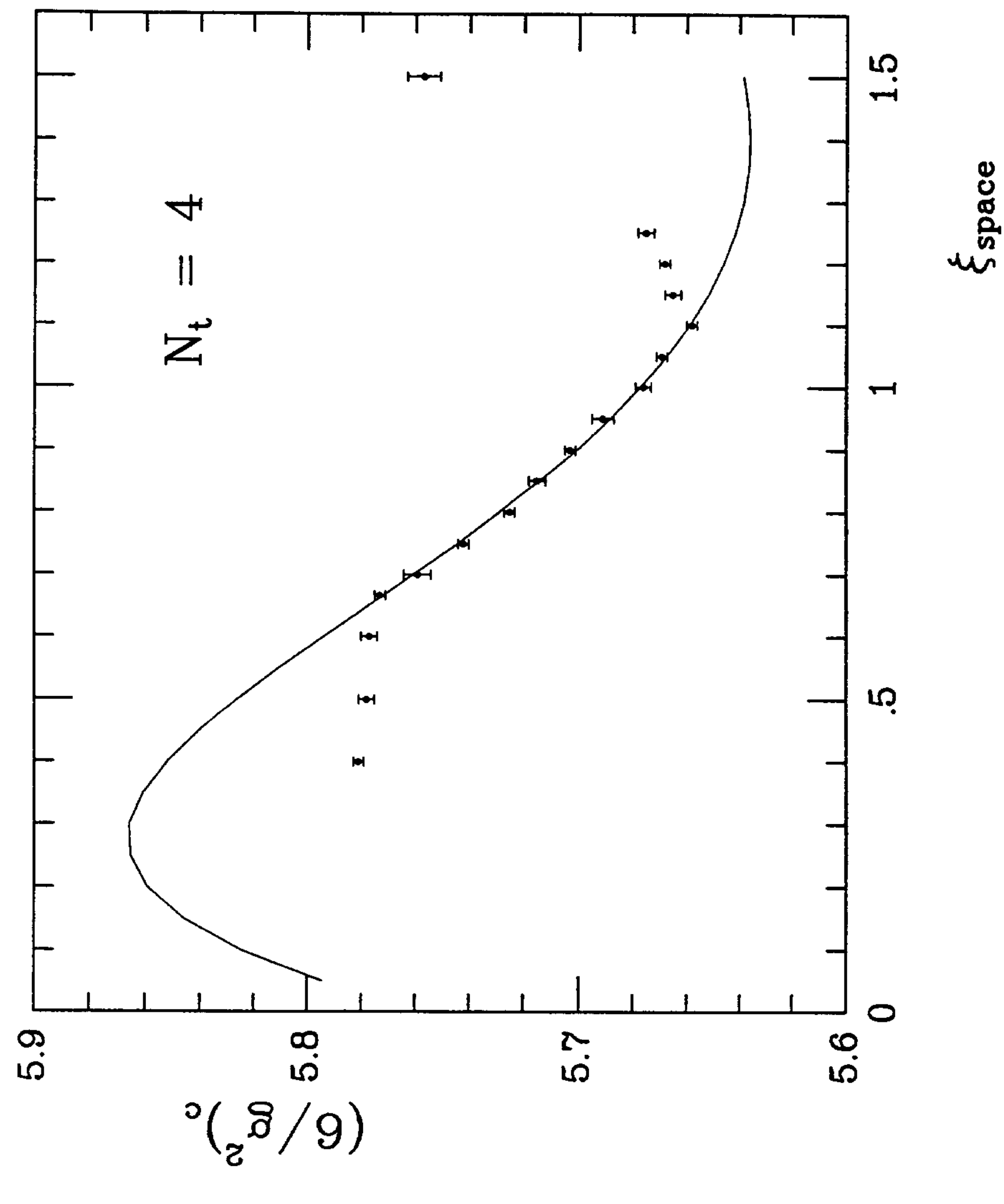

Figure 3